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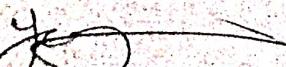
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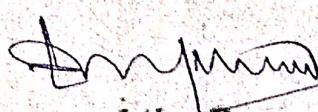
Advanced Numerical Analysis

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Adharsh Nagarjuna University.


Project Coordinator


In charge of the Department.

Dkr.
EXTERNAL EXAMINER

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* Numerical Analysis *

Numerical Analysis is the study of algorithms that use numerical approximation for the purpose of mathematical analysis.

Numerical Analysis naturally finds application in all fields of engineering and physical sciences but in the 21st century also the life sciences, social sciences, medicine, business and even the arts have adopted elements of scientific computations. The growth in computing power has revolutionized the use of realistic mathematical models in science and engineering and subtle numerical analysis is required to implement these detailed models of the world.

* For example ordinary differential equations appear in celestial mechanics.

* Numerical Linear Algebra is important for Data Analysis

- * Stochastic differential equations and markov chains are essential in simulating living cells for medicine and biology
- * Numerical analysis continues this long tradition
 - Oh rather than exact symbolic answers, which can only be applied to real world measurements by translation into digits
- * It gives approximate solutions within specified error bounds.

Introduction :-

- The overall goal of the field of numerical analysis is the design and analysis of techniques to give approximate but accurate solutions to hard problems.
- * Advanced numerical methods are essential in making "Numerical weather prediction" feasible.
 - * Computing the trajectory of a space craft requires the accurate numerical solution of a

system of ordinary differential equations.

- * car companies can improve the crash safety of their vehicles by using computer simulations of car crashes. such simulations essentially consist of solving partial differential equations numerically
- * Insurance companies use numerical programs for actual analysis.

Generation and propagation of Errors

The study of error forms an important part of numerical analysis can be introduced in the solution of the problem.

- Round off
- Truncation and discretization error
- Numerical stability and well-posed problems
- Interpolation, Extrapolation and regression
- Solving Equations and system of Equations
- Solving eigen values (or) singular value problems.
- optimization

Problem 1

Title :- Laplace transform of $\int_0^t \left(\frac{1-e^{-2x}}{x} \right) dx$

Aim :- To find Laplace transform of

$$\int_0^t \left(\frac{1-e^{-2x}}{x} \right) dx$$

Formula :- $L\{1-e^{-2x}\} = \frac{1}{p} - \frac{1}{p+2}$

Detailed solution :-

$$f(x) = 1-e^{-2x}$$

$$\text{then } f(p) = L\{1-e^{-2x}\} = \frac{1}{p} - \frac{1}{p+2}$$

$$\therefore L\left\{\frac{1-e^{-2x}}{x}\right\} = \int_p^\infty \left(\frac{1}{p} - \frac{1}{p+2} \right) dp$$

$$= \left[\log p - \log(p+2) \right]_p^\infty$$

$$= \left[\log \left(\frac{p}{p+2} \right) \right]_p^\infty$$

$$= \left[\log \left(\frac{1}{1+2/p} \right) \right]_p^\infty$$

$$= \log(0) - \log \left(\frac{p}{p+2} \right)$$

$$= \log \left(\frac{p+2}{p} \right) = f(p)$$

$$\Rightarrow \log\left(\frac{p+2}{p}\right) = f(p)$$

$$\therefore L\left\{\int_0^t \left(1 - \frac{e^{-2x}}{x}\right) dx\right\} = \frac{1}{p} f(p)$$
$$= \frac{1}{p} \log\left(\frac{p+2}{p}\right)$$
$$= \frac{1}{p} \log\left(1 + \frac{2}{p}\right)$$

Conclusion :-

Laplace transform of

$$\int_0^t \left(1 - \frac{e^{-2x}}{x}\right) dx = \frac{1}{p} \log\left(1 + \frac{2}{p}\right)$$

Problem 2

Title :- Initial value theorem and final value theorem

Aim :- To prove initial value theorem and final value theorem

$$\text{① } \underset{t \rightarrow 0}{\text{Lt}} f(t) = \underset{s \rightarrow \infty}{\text{Lt}} s f(s)$$

$$\text{② } \underset{t \rightarrow \infty}{\text{Lt}} f(t) = \underset{s \rightarrow 0}{\text{Lt}} s f(s)$$

Detailed solution :-

(i) we know that

$$\mathcal{L}\{F'(t)\} = s f(s) - F(0)$$

$$\Rightarrow \int_0^\infty F'(t) e^{-st} dt = s f(s) - F(0)$$

As $s \rightarrow \infty$

$$\underset{s \rightarrow \infty}{\text{Lt}} \int_0^\infty e^{-st} F'(t) dt = \underset{s \rightarrow \infty}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow \int_0^\infty \underset{s \rightarrow \infty}{\text{Lt}} e^{-st} F'(t) dt + F(0) = \underset{s \rightarrow \infty}{\text{Lt}} s f(s)$$

$$\Rightarrow \int_0^\infty 0 \cdot F'(t) dt + F(0) = \underset{s \rightarrow \infty}{\text{Lt}} s f(s)$$

$$\Rightarrow F(0) = \underset{s \rightarrow \infty}{\text{Lt}} s f(s)$$

$$\Rightarrow \underset{t \rightarrow 0}{\text{Lt}} F(t) = \underset{s \rightarrow \infty}{\text{Lt}} s f(s)$$

This is called initial value theorem

(ii) we know that

$$\mathcal{L} \{ F'(t) \} = s f(s) - F(0)$$

$$\Rightarrow \int_0^\infty F'(t) e^{-st} dt = s f(s) - F(0)$$

As $s \rightarrow 0$

$$\underset{s \rightarrow 0}{\text{Lt}} \int_0^\infty e^{-st} F'(t) dt = \underset{s \rightarrow 0}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow \int_0^\infty F'(t) dt = \underset{s \rightarrow 0}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow \int_0^\infty \frac{dF}{dt} dt = \underset{s \rightarrow 0}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow [F(t)]_0^\infty = \underset{s \rightarrow 0}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow \underset{t \rightarrow \infty}{\text{Lt}} F(t) - F(0) = \underset{s \rightarrow 0}{\text{Lt}} s f(s) - F(0)$$

$$\Rightarrow \underset{t \rightarrow \infty}{\text{Lt}} F(t) = \underset{s \rightarrow 0}{\text{Lt}} s f(s)$$

this is called final value theorem

Conclusion :-

$$(i) \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s F(s)$$

$$(ii) \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s)$$

Problem 3

Title :- Inverse Laplace transform of $\mathcal{L}^{-1} \left[\frac{P}{(P+3)^{7/2}} \right]$

Aim :- To find inverse Laplace transform of

$$\mathcal{L}^{-1} \left[\frac{P}{(P+3)^{7/2}} \right]$$

Detailed solution :-

$$\begin{aligned}
\mathcal{L}^{-1} \left[\frac{P}{(P+3)^{7/2}} \right] &= \mathcal{L}^{-1} \left\{ \frac{P+3-3}{(P+3)^{7/2}} \right\} \\
&= e^{-3t} \mathcal{L}^{-1} \left\{ \frac{P-3}{P^{7/2}} \right\} \quad \text{by FST} \\
&= e^{-3t} \mathcal{L}^{-1} \left\{ \frac{1}{P^{5/2}} - \frac{3}{P^{7/2}} \right\} \\
&= e^{-3t} \mathcal{L}^{-1} \left\{ \frac{1}{P^{3/2+1}} - 3 \cdot \frac{1}{P^{5/2+1}} \right\} \\
&= e^{-3t} \left[\frac{t^{3/2}}{\Gamma(3/2+1)} - 3 \cdot \frac{t^{5/2}}{\Gamma(5/2+1)} \right] \\
&= e^{-3t} \left[\frac{t^{3/2}}{\frac{1}{2}\sqrt{\pi}} - 3 \cdot \frac{t^{5/2}}{\frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\frac{1}{2}}} \right] \\
&= \frac{e^{-3t}}{\sqrt{\pi}} \left[\frac{4}{3} t^{3/2} - \frac{8}{5} t^{5/2} \right]
\end{aligned}$$

Conclusion :-

$$\mathcal{L}^{-1} \left\{ \frac{P}{(P+3)^{7/2}} \right\} = \frac{e^{-3t}}{\sqrt{\pi}} \left[\frac{4}{3} t^{3/2} - \frac{8}{5} t^{5/2} \right]$$

problem 4

Title :- Inverse Laplace transform of $L^{-1}\left\{\frac{P^2}{(P^2+4)^2}\right\}$

Aim :- To find inverse Laplace transform of

$$L^{-1}\left\{\frac{P^2}{(P^2+4)^2}\right\}$$

Detailed solution :-

$$\frac{P^2}{(P^2+4)^2} = \frac{P^2}{(P^2+4)} \cdot \frac{P}{(P^2+4)}$$

let $f(p) = \frac{P}{P^2+4}$ then

$$L^{-1}\{f(p)\} = L^{-1}\left\{\frac{P}{P^2+4}\right\} = \cos 2t = F(t)$$

using convolution theorem

$$\begin{aligned} L^{-1}\{f(p)f(p)\} &= \int_0^t f(u) F(t-u) du \\ &= \int_0^t \cos 2u \cos 2(t-u) du \\ &= \frac{1}{2} \int_0^t 2 \cos 2u \cos 2(t-u) du \\ &= \frac{1}{2} \int_0^t [\cos 2t + \cos 2(t-2u)] du \\ &= \frac{1}{2} \left[u \cos 2t + \frac{\sin 2(t-2u)}{-4} \right]_{u=0}^t \end{aligned}$$

$$= \frac{1}{2} \left(t \cos 2t - \frac{\sin 2(-t)}{4} + \frac{\sin 2t}{4} \right)$$

$$= \frac{1}{2} \left(t \cos 2t + \frac{2 \sin 2t}{4} \right)$$

$$= \frac{1}{2} (t \cos 2t + \frac{1}{2} \sin 2t)$$

Conclusion :-

$$\mathcal{L}^{-1} \left\{ \frac{p^2}{(p^2+4)^2} \right\} = \frac{1}{2} (t \cos 2t + \frac{1}{2} \sin 2t)$$

problem 5 :

Title :- solve $ty'' + y' + 4ty = 0$
 $y(0) = 3, y'(0) = 0$

Aim :- TO solve

$$ty'' + y' + 4ty = 0 \text{ at } y(0) = 3, y'(0) = 0$$

Detailed solution :-

Given,

$$ty'' + y' + 4ty = 0$$

taking Laplace transform on both sides

$$\mathcal{L}\{ty'' + y' + 4ty\} = 0$$

$$\Rightarrow \mathcal{L}\{ty''\} + \mathcal{L}\{y'\} + 4\mathcal{L}\{ty\} = 0$$

$$\Rightarrow -\frac{d}{dp} \mathcal{L}\{y''\} + \mathcal{L}\{y'\} + 4(-1)\frac{d}{dp} \mathcal{L}\{y\} = 0$$

$$\Rightarrow -\frac{d}{dp} [p^2 \mathcal{L}\{y\} - py(0) - y'(0)] + [p \mathcal{L}\{y\} - y(0)] - 4 \frac{d}{dp} \mathcal{L}\{y\} = 0$$

$$\Rightarrow -\frac{d}{dp} (p^2 z - 3p) + pz - 3 - 4 \frac{dz}{dp} = 0$$

where $\mathcal{L}\{y\} = z$

$$\Rightarrow -p^2 \frac{dz}{dp} - 2pz + 3 + pz - 3 - 4 \frac{dz}{dp} = 0$$

$$\Rightarrow -(p^2 + 4) \frac{dz}{dp} - pz = 0$$

$$\Rightarrow \frac{dz}{dp} + \frac{p}{(p^2 + 4)} z = 0$$

Integrating

$$\Rightarrow \log z + \frac{1}{2} \log(p^2 + 4) = \log C$$

$$\Rightarrow \log(z(p^2 + 4)^{1/2}) = \log C$$

$$\Rightarrow z = \frac{C}{\sqrt{p^2 + 4}}$$

$$\Rightarrow y = C \left[\frac{1}{\sqrt{p^2 + 4}} \right]$$

$$\therefore y = C \left[\frac{1}{\sqrt{t^2 + 4}} \right]$$

$$\Rightarrow y = C J_0(2t)$$

$$y(0) = 3 \Rightarrow 3 = C J_0(0) \\ = C$$

\therefore the required solution is $y = 3 J_0(2t)$

Conclusion :-

solution of $t y'' + y' + 4ty = 0$ at $y(0) = 3$,

$y'(0) = 0$ is

$$y = 3 J_0(2t)$$

problem 6:

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Title :- solve $(D^2 + 5D + 6)y = 5e^t$
 $y(0) = 2, y'(0) = 1$ when $t = 0$

Aim :- TO solve $(D^2 + 5D + 6)y = 5e^t$

at $y(0) = 2, y'(0) = 1$ when $t = 0$

Detailed solution :-

Given differential equation is

$$y'' + 5y' + 6y = 5e^t \rightarrow ①$$

Given conditions are $y(0) = 2, y'(0) = 1$

when $t = 0$

By taking Laplace transforms on both sides
of Eqn ①

$$\mathcal{L}\{y''\} + 5\mathcal{L}\{y'\} + 6\mathcal{L}\{y\} = 5\mathcal{L}\{e^t\}$$

$$\Rightarrow [p^2 \mathcal{L}\{y\} - py(0) - y'(0)] + 5[p\mathcal{L}\{y\} - y(0)] + 6\mathcal{L}\{y\} = \frac{5}{p-1}$$

$$\Rightarrow \mathcal{L}\{y\} [p^2 + 5p + 6] - 2p - 1 - 10 = \frac{5}{p-1}$$

$$\Rightarrow \mathcal{L}\{y\} [p^2 + 5p + 6] = \frac{5}{p-1} + 2p + 11$$

$$\Rightarrow \mathcal{L}\{y\} = \frac{5}{(p-1)(p+2)(p+3)} + \frac{2p}{(p+2)(p+3)} + \frac{4}{(p+2)(p+3)}$$

$$= \frac{2p^2 + 9p - 6}{(p-1)(p+2)(p+3)}$$

by applying partial fractions

$$\mathcal{L}\{y\} = \frac{5}{12} \left(\frac{1}{p-1} \right) + \frac{16}{3} \left(\frac{1}{p+2} \right) - \frac{15}{4} \left(\frac{1}{p+3} \right)$$

$$y = \frac{5}{12} \mathcal{L}^{-1} \left(\frac{1}{p-1} \right) + \frac{16}{3} \mathcal{L}^{-1} \left(\frac{1}{p+2} \right) - \frac{15}{4} \mathcal{L}^{-1} \left(\frac{1}{p+3} \right)$$

$$= \frac{5}{12} e^t + \frac{16}{3} e^{-2t} - \frac{15}{4} e^{-3t}$$

Conclusion :-

The solution for the given differential equation is

$$y = \frac{5}{12} e^t + \frac{16}{3} e^{-2t} - \frac{15}{4} e^{-3t}$$

Problem 7

Title :- Solve $(D-2)x - (D+1)y = 6e^{3t}$

$$(2D-3)x + (D-3)y = 6e^{3t} \text{ if } x(0)=3, y(0)=0$$

Aim :- To solve $(D-2)x - (D+1)y = 6e^{3t}$

$$(2D-3)x + (D-3)y = 6e^{3t}$$

$$\text{if } x(0)=3, y(0)=0$$

Detailed Solution :-

Given solutions are

$$(D-2)x - (D+1)y = 6e^{3t}$$

$$(2D-3)x + (D-3)y = 6e^{3t}$$

$$\Rightarrow x' - 2x - y' - y = 6e^{3t}$$

$$\text{and } 2x' - 3x + y' - 3y = 6e^{3t}$$

Taking Laplace transform on both sides

$$L\{x'\} - 2L\{x\} - L\{y'\} - L\{y\} = 6L\{e^{3t}\}$$

$$\text{and } 2L\{x'\} - 3L\{x\} + L\{y'\} - 3L\{y\} = 6L\{e^{3t}\}$$

$$\Rightarrow \{p\bar{x} - x(0)\} - 2\bar{x} - \{p\bar{y} - y(0)\} - \bar{y} = \frac{6}{p-3}$$

$$\therefore L\{x\} = \bar{x} \text{ and } L\{y\} = \bar{y}$$

$$\text{and } 2L\{x'\} = 3L\{x\} + L\{y'\} - 3L\{y\} \\ = 6L\{e^{3t}\}$$

$$\Rightarrow \{p\bar{x} - x(0)\} - 2\bar{x} - \{p\bar{y} - y(0)\} - \bar{y} = \frac{6}{p-3}$$

$$\therefore L\{x\} = \bar{x} \text{ and } L\{y\} = \bar{y}$$

$$\Rightarrow (p-2)\bar{x} - (p+1)\bar{y} = 3 + \frac{6}{p-3} \\ = \frac{3p-3}{p-3} \rightarrow \textcircled{1}$$

$$\text{and } (2p-3)\bar{x} + (p-3)\bar{y} = \frac{6p-12}{p-3} \rightarrow \textcircled{2}$$

$$\therefore (p-3) \text{ Eqn } \textcircled{1} + (p+1) \text{ Eqn } \textcircled{2}$$

$$\Rightarrow [(p-3)(p-2)(0) + (p+1)(2p-3)]\bar{x} \\ = 3p-3 + \frac{(p+1)(6p-12)}{p-3}$$

$$\Rightarrow (3p^2 + 6p + 3)\bar{x} = \frac{3p^2 - 12p + 9 + 6p^2 - 6p - 12}{p-3} \\ = \frac{9p^2 - 18p - 3}{p-3}$$

$$3(p-1)^2 \bar{x} = \frac{3(3p^2 - 6p - 1)}{p-3}$$

$$\Rightarrow L\{x\} = \frac{3p^2 - 6p - 1}{(p-1)^2(p-3)}$$

$$\Rightarrow L\{x\} = \frac{1}{(p-1)} + \frac{2}{(p-1)^2} + \frac{2}{(p-3)} \rightarrow ②$$

Find $(p-2)$ eqn ② - $(2p-3)$ eqn ①

$$\Rightarrow [(p-2)(p-3) + (2p-3)(p+1)] \bar{y} = \frac{6(p-2)^2}{p-3} - \frac{3(p-1)(2p-3)}{p-3}$$

$$\Rightarrow (3p^2 - 6p + 3) \bar{y} = -9p + 15$$

$$\Rightarrow L\{y\} = \frac{-3p + 5}{(p-3)(p+1)^2}$$

$$= \frac{1}{(p-1)} - \frac{1}{(p-1)^2} - \frac{1}{(p-3)} \rightarrow ④$$

From ③

$$x = L^{-1} \left[\frac{1}{p-1} \right] + 2L^{-1} \left[\frac{1}{(p-1)^2} \right] + 2L^{-1} \left[\frac{2}{p-3} \right]$$

$$= e^t + 2te^t + 2e^{3t}$$

From ④

$$y = L^{-1}\left(\frac{1}{p-1}\right) - L^{-1}\left(\frac{1}{(p-1)^2}\right) - L^{-1}\left(\frac{1}{p-3}\right)$$
$$= e^{t} - t e^t - e^{3t}$$

Conclusion :-

The required solution for the given equations
are

$$x = e^t + 2t e^t + 2e^{3t}$$

$$y = e^t - t e^t - e^{3t}$$

Problem 8 *

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Title :- Method of finding fourier transform
of given function

Aim :- Find the fourier transform to the given
function.

Statement of the problem

Find the fourier transform $f(x)$ defined by

$$f(x) = \begin{cases} 1 & |x| > a \\ 0 & |x| < a \end{cases}$$

Hence prove that

$$\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$$

Formula and detailed solution

$$F(f(x)) = \bar{F}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} f(x) dx$$

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\bar{F}(p)|^2 dp$$

Detailed solution

$$F\{f(x)\} = \bar{F}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} f(x) dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} e^{ipx} f(x) dx + \int_{-\infty}^{\infty} e^{ipx} \bar{f}(x) dx + \int_{-\infty}^{\infty} e^{ipx} f(x) dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + \int_{-a}^a e^{ipx} f(x) dx + 0 \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-a}^a e^{ipx} f(x) dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{ipa}}{ip} \right]_{-a}^a$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{ip} [e^{ipa} - e^{-ipa}]$$

$$= \frac{\sqrt{2}}{\sqrt{\pi} p} \left(\frac{e^{ipa} - e^{-ipa}}{2i} \right)$$

$$F\{f(x)\} = \sqrt{\frac{2}{\pi}} \cdot \frac{\sin ap}{p}$$

using Parseval's Identity

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\bar{f}(p)|^2 dp$$

$$\Rightarrow \int_{-a}^a |f(x)|^2 dx = \int_{-\infty}^{\infty} \frac{2}{\pi} \cdot \frac{1}{p^2} \sin^2 ap \cdot dp$$

$$2a = 2 \cdot \frac{2}{\pi} \int_0^{\infty} \frac{\sin^2 ap}{p^2} dp$$

$$\therefore \int_0^{\infty} \frac{\sin^2 ap}{p^2} dp = \frac{2\pi a}{4}$$

$$\Rightarrow \int_0^{\infty} \frac{\sin^2 ap}{p^2} dp = \frac{\pi a}{2}$$

Conclusion :-

Solution of the fourier transform equation

$$\int_0^{\infty} \frac{\sin^2 ap}{p^2} dp = \frac{\pi a}{2}$$

problem 9

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Title :- using method of Least squares fit a straight line to the following data

x	1	2	3	4	5
y	2	4	6	8	10

Aim :- To fit a straight line for the given data by using the method of least square

Detailed solution :-

x_i	y_i	x_i^2	$x_i y_i$
1	2	1	2
2	4	4	8
3	6	9	18
4	8	16	32
5	10	25	50

From the table we have

$$n = 5, \sum x_i = 15, \sum y_i = 30, \sum x_i^2 = 55$$

$$\sum x_i y_i = 110$$

The normal Equations are

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$$na + b \sum x_i = \sum y_i$$

$$a \sum x_i + b \sum x_i^2 = \sum x_i y_i$$

$$\Rightarrow 5a + 15b = 30 \rightarrow ①$$

$$15a + 55b = 110 \rightarrow ②$$

$$① \times 15 \Rightarrow 75a + 225b = 450$$

$$② \times 5 \Rightarrow \underline{75a + 275b = 550}$$

$$- 50b = -100$$

$$\boxed{b = 2}$$

$$5a + 30 = 30$$

$$\Rightarrow \boxed{a = 0}$$

Conclusion :-

The straight line for the given data is

$$\boxed{y = 2x}$$

Problem 10

Title :- using the following table to compute

$$\frac{dy}{dx} \text{ and } \frac{d^2y}{dx^2} \text{ at } x=1$$

x	1	2	3	4	5	6
y	1	8	27	64	125	216

Aim :- To compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1$

for the given Data

Detailed solution :-

Given Data

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1	7			
2	8	12	6		
3	27	18	6	0	
4	64	24	6	0	
5	125	30	6		
6	216	91			

Here $x_0 = 1$, $n = 1$

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By Newton's forward interpolation formula

$$\left(\frac{dy}{dx} \right)_{x=x_0} = \frac{1}{h} \left[\Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \frac{1}{4} \Delta^4 y_0 + \dots \right]$$

$$\begin{aligned} \left(\frac{dy}{dx} \right)_{x=1} &= \frac{1}{1} \left[7 - \frac{1}{2} \times 12 + \frac{1}{3} \times 6 - \frac{1}{4} \times 0 \right] \\ &= [7 - 6 + 2] = 3 \end{aligned}$$

$$\begin{aligned} \left(\frac{d^2y}{dx^2} \right)_{x=1} &= \frac{1}{h^2} \left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 \right] \\ &= \frac{1}{(1)^2} (12 - 6) = 6 \end{aligned}$$

Conclusion :-

$$\left(\frac{dy}{dx} \right)_{x=1} = 3$$

$$\left(\frac{d^2y}{dx^2} \right)_{x=1} = 6$$

Problem 11

Title :- Find the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$
by using Simpson's $1/3$ & $3/8$ rule and
Hence obtain the approximate value of π
in each case.

Aim :- To find the value of $\int_0^1 \frac{1}{1+x^2} dx$ by using
Simpson's $1/3$ & $3/8$ rule and hence obtain
the approximate value of π in
each case.

Formulae :-

By Simpson's $1/3$ rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3} [y_0 + 4(y_1+y_2+y_3) + 2(y_2+y_4) + y_6]$$

By Simpson's $3/8$ rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8} [(y_0+y_6) + 3(y_1+y_2+y_4+y_6) + 2y_3]$$

Detailed solution :-

Divide $[0,1]$ into 6 equal sub-intervals

$$\Rightarrow h = \frac{1}{6}$$

$$\therefore y = f(x) = \frac{1}{4x^2}$$

x	0	y_6	y_3	y_2	y_1	y_0	1
y	1	0.9729	0.90	0.80	0.6923	0.5901	0.52
		y_6	y_3	y_2	y_1	y_0	y_5

By Simpson's $\frac{1}{3}$ rule

$$\begin{aligned}
 \int_0^1 \frac{dx}{1+x^2} &= \frac{h}{3} \left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \right] \\
 &= \frac{1}{18} \left[(1+0.5) + 4(0.9729 + 0.80 + 0.5901) + 2(0.90 + 0.6923) \right] \\
 &= \frac{1}{18} [1.5 + 9.952 + 3.1846] \\
 &= 0.785366
 \end{aligned}$$

By Simpson's $\frac{3}{8}$ rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8} \left[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2y_3 \right]$$

$$\begin{aligned}
 &= \frac{1}{16} \left[(1+0.5) + 3(0.9729 + 0.90 + 0.6923 + \right. \\
 &\quad \left. 0.5901) + 2(0.80) \right] \\
 &= \frac{1}{16} [1.50 + 3(3.1553) + 1.6] \\
 &= 0.785368
 \end{aligned}$$

But $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}(x)]_0^1 = \pi/4$

In case of Simpson's $1/3$ rule

$$\pi/4 = 0.785366 \Rightarrow \pi = 3.141464$$

In case of Simpson's $3/8$ rule

$$\pi/4 = 0.785368 \Rightarrow \pi = 3.141472$$

Conclusion :-

In case of Simpson's $1/3$ rule

$$\int_0^1 \frac{dx}{1+x^2} = 0.785366 \Rightarrow \pi = 3.141464$$

In case of Simpson's $3/8$ rule

$$\int_0^1 \frac{dx}{1+x^2} = 0.785368 \Rightarrow \pi = 3.141472$$

problem 12
www m

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Title :- solve

$$2x + y + z = 1, x - 2y - 3z = 1$$

$3x + 2y + 4z = 5$ by Gauss Jordan
method

Aim :- To solve given equations by Gauss
Jordan - method

Detailed solution :-

Given Equations are

$$2x + y + z = 1$$

$$x - 2y - 3z = 1$$

$$3x + 2y + 4z = 5$$

∴ The Augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -2 & -3 & 1 \\ 3 & 2 & 4 & 5 \end{array} \right]$$

$$R_2 \rightarrow 2R_2 - R_1, R_3 \rightarrow 2R_3 - 3R_1$$

$$2 \left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -5 & -7 & 1 \\ 0 & 1 & 5 & 7 \end{array} \right]$$

$$R_1 \rightarrow 5R_1 + R_2, \quad R_3 \rightarrow 5R_3 + R_2$$

2

$$\begin{bmatrix} 10 & 0 & -2 & 6 \\ 0 & -5 & 7 & 1 \\ 0 & 0 & 18 & 36 \end{bmatrix}$$

$$R_1 \rightarrow 9R_1 + R_3, \quad R_2 \rightarrow 18R_2 + 7R_3$$

2

$$\begin{bmatrix} 90 & 0 & 0 & 90 \\ 0 & -90 & 0 & 270 \\ 0 & 0 & 18 & 36 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

∴ By Gauss Jordan method

$$x=1, y=-3, z=2$$

Conclusion :-

The solutions for the given Equations are

$$x=1, y=-3, z=2$$

problem 13

title :- solve $\frac{dy}{dx} = (x+y)$, $y(1) = 0$ upto $x=1.2$

with $h=0.1$ using Taylor's series method

Aim :- to solve $\frac{dy}{dx} = (x+y)$

$y(1) = 0$ upto $x=1.2$ with $h=0.1$

Detailed solution :-

Taylor's series for $y(x)$ is given by

$$y = 1 + xy_0 + \frac{x^2}{2!} y_0'' + \frac{x^3}{3!} y_0''' + \dots$$

From the data we have

$$y^1 = x+y \Rightarrow y^{1(1)} = 1+0$$

$$y^{11} = 1+y^1 \Rightarrow y^{11(1)} = 1+1 = 2$$

$$y^{111} = 0+y^{11} = 0+2 \Rightarrow y^{111(1)} = 2$$

$$\therefore y(x) = 1 + x + \frac{2x^2}{2!} + \frac{2x^3}{3!} + \frac{2x^4}{4!} + \dots$$

Putting $x=1.1$

$$\text{we get } y(1.1) = 1 + 1 \cdot 1 + (1 \cdot 1)^2 + \frac{1}{3} (1 \cdot 1)^3 + \dots \\ = 1 + 1 \cdot 1 + 1 \cdot 21 + 0.4437 + \dots$$

For $x = 1.1$ and $y = 3.8757$ we have

$$y^1 = x+y = g^1(1.1) = 1+3.8757 = 4.9757$$

$$y^{11} \doteq 1+y^1 \Rightarrow y^{11}(1.1) = 1+4.9757 = 5.9787$$

$$y^{111} = 0+y^{11} \Rightarrow y^{111}(1.1) = 5.9787$$

$$\therefore y(x) = 1+x(4.9757) + \frac{x^2}{2!}(5.9787) + \dots$$

put $x = 1.2$ we get

$$y(1.2) = 1+(1.2)(4.9757) + \frac{x^2}{2!}(5.9787) + \frac{x^3}{3!}(5.9757)^2 + \dots$$

$$= 1+5.97084 + 4.3025 + 1.7210$$

$$= 12.9943$$

Conclusion :-

The solution of Given Equation is

$$y(1.2) = 12.9943$$